Business PreCalculus MATH 1643 Section 004, Spring 2014
Lesson 23: Exponential Functions

Definition 1. **Exponential Functions:** *A function f of the form*

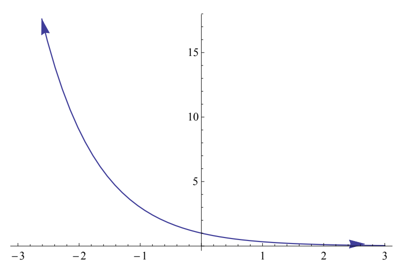
$$f(x) = a^x,$$

where $a > 0$ and $a \neq 1$ is called an ***exponential function with base*** *a. Its domain is $(-\infty, \infty)$.*

Definition 2. **Properties of Exponential Functions:** *Let $f(x) = a^x$, $a > 0$ and $a \neq 1$.*

1. *The domain of $f(x) = a^x$ is $(-\infty, \infty)$.*
2. *The range of $f(x) = a^x$ is $(0, \infty)$, which means that the entire graph lies above the x-axis.*
3. *For $a > 1$,*
   (a) *As $x \to \infty$, then $f(x) \to \infty$.*
   (b) *As $x \to -\infty$, then $f(x) \to 0$, which means that $y = 0$ is the horizontal asymptote.*
4. *For $0 < a < 1$,*
   (a) *As $x \to -\infty$, then $f(x) \to \infty$.*
   (b) *As $x \to \infty$, then $f(x) \to 0$, which means that $y = 0$ is the horizontal asymptote.*
5. *The graph of $f(x) = a^x$ has no x-intercepts.*

Example 1. *The graph of $f(x) = (\frac{1}{3})^x$ is*



Example 2. *Let $f(x) = 3^{x-2}$ be an exponential function. Find $f(4)$ and $f(2)$.*
Solution: *Since $f(x) = 3^{x-2}$, then*

$$f(4) = 3^{4-2} = 3^2 = 9.$$

and

$$f(2) = 3^{2-2} = 3^0 = 1.$$

Definition 3. **Exponential Equations:** *In case of same base a, the equation*

$$a^u = a^v \text{ implies } u = v.$$

Example 3. *Solve the exponential equations* $2^x = 128$ *and* $5^{x(x-3)} = \frac{1}{25}$ *for* x.

Solution: *For the equation* $2^x = 128$,

$$\begin{aligned} 2^x &= 128 \\ 2^x &= 2^6 \end{aligned}$$

Then $x = 6$.

For the equation $5^{x(x-3)} = \frac{1}{25}$,

$$\begin{aligned} 5^{x(x-3)} &= \frac{1}{25} \\ 5^{x(x-3)} &= 5^{-2} \end{aligned}$$

Then

$$\begin{aligned} x(x-3) &= -2 \\ x^2 - 3x &= -2 \\ x^2 - 3x + 2 &= 0 \\ (x-2)(x-1) &= 0 \end{aligned}$$

Then $x = 2$ *or* $x = 1$.

Example 4. *Find the exponential function* $f(x) = a^x$ *that contains* $(3, 125)$.

Solution: *Since* $f(x)$ *contains* $(3, 125)$, *then* $125 = a^3$ *which implies that* $a = 5$. *Hence,* $f(x) = 5^x$ *for any* x.